Experimenting “fisheye-lens functions” in studying digitally particular historic maps

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**Summary**

Historical maps are rich in the way their cartographic content is represented both in the geometric – projection component of the representation and in the thematic counterpart. Focusing into the geometric content of old maps, we can identify exceptional examples of maps designed in particular projections of non-conventional types, as they are e.g. the bird-eye view projections related, in general, to the perspective representation, or the more complicated fisheye view projections, which is the very interesting case of some rare and thus important old city maps. Dealing with the second case of representations, the projection properties are not known and the only evidence which could assist the analysis is based on an initially intuitive approach, associated to the phenomenological assimilation of the map geometry pattern. The analysis can only be based on a test-and-trial procedure, i.e., by comparing the degree of agreement of the original map with models developed using relevant mapping functions which are generally known in the modern photographic image capturing literature as fisheye-lens functions. In this study the famous city map of Argentoratum (Strasbourg) is used, in digital form, taken from Braun and Hogenberg’s *Civitates Orbis Terrarum* (1572) in order to study the apparent non linear projection, of a fisheye view type, of this map. A digital analysis using types of appropriate fisheye lens functions, combining cartographic and photogrammetric methods, shows the projection type inherent in the map representation, in association to a comparative study using relevant modern maps in regular projections.

**Introduction**

In the history of cartography and maps, especially of the 16th century onwards, the representations of cities and towns are very important. The case of the fundamental publication *Civitates Orbis Terrarum* by Braun and Hogenberg is very well known and referential. In the various editions of this publication, up to the first half of 17th century, almost 300 urban representations are presented. Among them there is the city of Argentoratum, the Latin name of Strasbourg, capital city of Alsace and a reference city of today’s Europe. Studying the images of all these 300 representations, only that of Argentoratum looks unique for its particular projection form in which the city is depicted. In Fig. 1, this map is shown taken from the 1572 Cologne edition of Braun and Hogenberg city-plan Atlas. No

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other city, among those present in this Atlas, is represented in the projection depicting the Alsatian capital. This particular projection, unique for Strasbourg, at least from the mid 16th century to the first quarter of the 17th century, is of a cartographic typology often called today focal, or logarithmic, or variable-scale, or lens, or even fisheye projection. This type of projection (Fig. 2a, Fig. 2b), with various names, is currently still used after some decades of implementation, since it looks to be very popular among younger researchers in applications of a variety of modern scientific fields.\(^1\)

In this paper the Braun and Hogenberg 1572 city map of Argentoratum is used, in digital form in order to study this not common projection, of a fisheye view type. A digital analysis, in association to a comparative study using relevant modern maps in regular projections, using types of appropriate functions and the test-and-trial procedure, very adequate in such cases,\(^2\) shows that the projection type inherent in the map representation under examination is much more complicated than it appears at first sight.

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\(^1\) See, e.g., Kadmon and Shlomi (1978); Boutoura (1994); Fairbairn and Taylor (1995); Boutoura et al. (1999); Guerra and Boutoura (2001); Harrie et al. (2002); Pérez and Andreu (2008); Yamamoto et al. (2009); Walsh et al. (2010); Haunert and Sering (2011); Wu and Liu (2011); Michalski and Tymków (2011). For a rough introduction in the fisheye-lens concept and the relevant projection, see: http://en.wikipedia.org/wiki/Fisheye_lens; http://wiki.panotools.org/Fisheye_Projection.

\(^2\) For a typical example of using the test-and-trial procedure in historical map projection analysis, see e.g., Boutoura (2006).
Figure 2a. Examples of the focal (mono-focal and poly-focal) representation of city-maps (here the intramural part of the city of Thessaloniki), after Boutoura, 1994. Some city parts with special interest are focused in (varying) grater scale without losing the spatial continuity of the city layout.

Figure 2b. Examples of the mono-focal representation of interactive city-image maps (here the San Marco Square of Venice), after Guerra and Boutoura, 2001.
The map under question and the supporting cartographic material

The 1572 map, under examination here, was preceded some decades before by the quite similar map of Argentoratum (Fig. 3), published in Strasbourg in 1548 by Conrad Morant, a native of Basel in the early 16th century (~1510), active in the Alsatian capital in the mid-sixteenth century. As shown in Fig. 4, the two maps belong to the same typology, as the finite element analysis indicates. Apparent differences are attributed mainly to the design of the content, concerning e.g. the edifices, the drawing of the cathedral and the overall aesthetics which characterize the visual perception of the two maps. The general pattern of the spatial layout as well as of the street network remains the same, in both maps. Here, it should be mentioned that the Argentoratum map in *Civitates Orbis Terrarum* is derived from the relevant map in the German edition of Ptolemy’s *Geographia*, 1550, by Sebastian Münster published (as Cosmographia) in Basel, the native city of Conrad Morant who, few years before, made his own map of Strasbourg. Apparently Morant has collaborated on Münster’s work.

![Figure 3. Argentoratum (Strasbourg) by Conrad Morant, 1548.](image)

*Source: Germanisches Nationalmuseum, Nürnberg; Arsenale Editice (1999), Venezia.*

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Figure 4. The finite element fitting of the 1548 and the 1572 maps of Argentoratum (see Figs. 1 and 2). The fitting shows, via the digital transparency overlapping, the overall pattern agreement of the spatial layout as well as the total coincidence of the street network.

Figure 5. A city map of central Strasbourg, dated 1888. This map, after being georeferenced, was used as the reference map of known projection in the test-and-trial analysis carried out in this study. Source: Wikipedia (version in English).
For the analysis carried out in this research a “modern” city map of central Strasbourg, from the late 19th century is used, obviously derived from land surveying measurements (Fig. 5). This “modern” map, dated 1888, after a proper georeference, was possible to be easily transformed in any map projection in order to perform the test-and-trial process followed in the comparative analysis. This lead to the identification of the projective properties, best approximated, of the particular mid-sixteen century representation of Strasbourg, of the fisheye-lens typology.

The test-and-trial processing

The “modern” 1888 map is used for the processing. This was done, using properly the Google Earth projective properties machinery. After transforming initially the map in the plate-carée projection we consequently transform it into the orthographic projection, which is nominally relevant to the fisheye-lens projection typology. The result of this transformation is shown in Fig. 5, adding on the image an arbitrary rectangular grid, useful for assisting the visualization in the test-and-trial process.

Figure 6. The 1888 Strasbourg map (1888) transformed in orthographic projection. The blue rectangular grid is chosen arbitrarily in order to support the test-and-trial process.

The map in orthographic projection is then fitted to the Civitates map; judging from the deformation of the auxiliary rectangular grid, it is evident that a perspective component is also inherent in the Civitates map after the fitting (Fig. 7).

5 The use of this 1888 map is due to its convenient date of issue and especially due to its accessibility in high resolution digital copy available in: http://en.wikipedia.org/wiki/File:Karte_Strassburg_MK1888.png (the Wikipedia English version).
Figure 7. The distortion of the auxiliary arbitrary grid shows the inherent perspectivity component in the *Civitates* map.

Omitting the perspective component, we obtain a perspective-free new image as shown in Fig. 8.

Figure 8. The perspective-free *Civitates* map.
In order to omit the perspective component, special photogrammetric software was applied. This software corrects the deformation due to the central projection induced by the camera lens, allowing manual rotational operations (Fig. 9) and thus fulfilling the requirement for a perspective-free new image.

The perspective-free image is then derived applying an optical correction caused by axial rotations up to $45^\circ$ with $1^\circ$ rotation step. The relevant transformation is:

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\begin{align*}
    x_u &= x_d + (x_d-x_c) \left( K_1 r^2 + K_2 r^4 + \ldots \right) + (P_1 \left( r^2 + 2(x_d-x_c)^2 \right) + 2P_2(x_d-x_c)(y_d-y_c)) \left( 1 + P_3 r^2 + \ldots \right) \\
    y_u &= y_d + (y_d-y_c) \left( K_1 r^2 + K_2 r^4 + \ldots \right) + (P_1 \left( r^2 + 2(y_d-y_c)^2 \right) + 2P_2(x_d-x_c)(y_d-y_c)) \left( 1 + P_3 r^2 + \ldots \right)
\end{align*}
$$

Then, we reiterate the process fitting the 1888 map image in orthographic projection (Fig. 6) onto the perspectivity-free map (Fig. 8) obtaining a new distortion of the arbitrary grid from which we observe a clear *fisheye-lens* effect, as it is shown in Fig. 10.

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6 The Pt-Lens software is used here. See http://epaperpress.com/ptlens/
Figure 10. The best fitting of the 1888 map in orthographic projection onto the perspectivity-free Civitates map shows clearly the fisheye-lens effect inherent in the original representation.

After absorbing the perspectivity the fisheye transformation was created using the Brown equations. A special algorithm was used, for the simulation of the deformation, using the OpenCV library for digital image processing. Modifications implemented properly in the software introduced the fisheye effect through a fourth order polynomial function of the distance of a given point from its focal origin. The focal origin is defined manually on the image and the definition of the order of the deformation is defined using a slider. The order of the focal deformation is defined as a coefficient of the power of 2 and 4 of the radius distance of the given point on the image from the focal origin (Fig. 11).
Comparative analysis

A next step is an attempt for a comparative analysis of the map content features, as they are e.g. the blocks of the edifices. This is done by digitizing the blocks on the following two projective images:

A. On the image of the 1888 map in orthographic projection after applying the best fitting onto the perspective-free Civitates map (Fig. 10, Fig. 12-Left). The digitization of the blocks in this case is shown in Fig. 12-Right.

![Figure 12. Left: The 1888 map in orthographic projection best fitted on the perspective-free Civitates map. Right: The digitization of the blocks (blue surfaces).](image)

B. On the image of the 1888 map in orthographic projection after applying the software introducing the fisheye effect. The result of this application is shown in Fig. 13-Left and the digitization of the blocks in Fig. 13-Right.

![Figure 13. Left: The result of applying the software introducing the fisheye effect on the 1888 map in orthographic projection of Fig. 6. Right: The digitization of the blocks (orange surfaces).](image)
The comparison of the deformed arbitrary grid according to the images of the above case A (blue grid) and B (red grid) is shown in Fig. 14-Left, whereas the comparison of the blocks spatial feature in the cases A (blue blocks) and B (orange blocks) is shown in Fig. 14-Right.

![Figure 14](image1.png)

**Figure 14.** Left: The comparison of the deformed grid due to the case A (blue) and the case B (red). Right: The comparison of the blocks spatial feature due to the case A (blue) and the case B (orange).

The previous comparisons are summarized with respect to the relevant map images in Fig. 15. In the left image it is shown the grid and the blocks referred to the case A fitted to the perspective-free *Civitates* map and in the right image it is shown the grid and the relevant blocks referred to the case B, fitted to the perspective-free *Civitates* map too.

![Figure 15](image2.png)

**Figure 15.** Left: The grid and the relevant blocks referred to the case A fitted to the perspectivity-free *Civitates* map (Fig. 8) Right: The grid and the relevant blocks referred to the case B fitted to the perspective-free *Civitates* map (Fig. 8)
Conclusions

The conclusions of this study can be summarized in the following general and specific points: The access to free images in the web today, in adequate resolution, which is increasing continuously in numbers and quality, gives new possibilities for analytic research on the content of important historic maps. This new fact on the accessibility of cartographic heritage material makes easier the acquisition of old maps in digital form. Digital analytical tools applied in the analysis of important unique pieces of cartographic heritage, as the example used and treated in this study, are powerful in providing results and insights which otherwise are impossible to draw by other means. Especially in complicated issues when it is not always enough to use the so called black-box analytic software tools suited for only one-way dedicated processes, the combination of more than one processing tools through test-and-trial approaches, enrich the analysis and offer alternatives and flexibility for approaching better the target of the study. In our case it was shown that methods and techniques used in digital Cartography can be fruitfully coupled and combined with analogous methods and techniques used in digital Photogrammetry, offering comparably close results.

Dealing now with the specific case of the Renaissance representation of the city of Argentoratum (Strasbourg), included as an unique item of a particular projection in the Atlas of city-plans of the mid-16th to early 17th century, the remarkable in the history of cartography Braun and Hogenberg’s Civitates Orbis Terrarum, our test-and-trial digital analysis shows that the projection induced in this representation is better fitted to a fisheye-lens projection when an inherently present spatial component of perspectivity is deducted from the image. This perspectivity effect was possible to be identified and removed only by applying the test-and-trial digital methods used in this study.

Having removed the perspectivity, the orthographic projection tests used showed a remarkable coherence in our analysis. The same came out from the photogrammetric approach used, relevant to the lens distortion correction applied in photo image reductions. As it is shown from the final fitting comparisons of the two approaches, introducing the orthographic projection intermediation and deducing the perspectivity-effect, the strictly cartographic and the photogrammetric approach converge in results which keep spatially constant the major features of the map, i.e. the main squares of the city remaining at the same spatial position as well as the course of the river surrounding the central part of the city.

References


Perspective projections are well known and familiar images in historic representations, especially of cities, are often called bird’s-eye views.


