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Visualization and GIS Analysis of Ptolemy's One-Sided Globe in the Old and Modern Contexts

Keywords: Claudius Ptolemy, ancient geography, GIS analysis, historical cartography, georeferencing

Summary: In this paper, we frame the results of our multi-year project devoted to georeferencing and visualization of data from Claudius Ptolemy's seminal 'Geography' in the context of other ancient, medieval and modern cartographic sources, some of which were influenced by or derived from Ptolemy's massive and widely venerated classical work. Our GIS analysis involves such relevant old sources as Konrad Miller's consolidated Tabula Rogeriana by Muhammad al-Idrisi, Tabula Peutingeriana, Hellenic Ptolemaic maps, and the Alfonsine tables. We also discuss the challenges of visualizing Ptolemy's one-sided globe of oikouménē, the 'known world' of the ancients, using such modern GIS tools for development of 3D virtual globes as Google Earth and Cesium.

Introduction

Claudius Ptolemy was a major Hellenic scholar universally recognized for his prominent role in establishing the foundations of astronomy and geography as scientific disciplines. He is believed to have lived in Alexandria, Egypt, in the 2nd century AD. Ptolemy's classic *Geography* (Stückelberger & Grasshoff, 2006) is a massive work providing numerical coordinates of 6300+ objects of *oikouménē*, the "known world" of the ancients. The set of objects comprises cities, towns, colonies, villages, markets, harbors, temples, altars, mountains, capes, bays, lakes, forests, river sources and mouths, etc. In addition to the objects with coordinates (longitudes and latitudes), Ptolemy offers useful descriptions and lists of tribes that once inhabited different parts, or provinces of the *oikouménē*, along with the information on which Roman legions were stationed in which cities, distance computations, reasoning about the shape and size of the "known world", and map-making instructions for three different projections. Livieratos (2006) stressed the modern need for "a rigorous revisiting of Ptolemy's representations, especially the regional tabulae, in terms of georeferencing."

In this paper, we frame the results of our multi-year project devoted to georeferencing, GIS analysis and visualization of data from Ptolemy's *Geography* in the context of other ancient, medieval and modern cartographic sources, some of which were influenced by or derived from Ptolemy's opus magnum. Our earlier publications concerned Ptolemy's Taprobane and India before the Ganges (Abshire et al. 2016), Arabia (Abshire et al. 2020), the Fertile Crescent including Judaea Palestina, Syria, Mesopotamia, and Babylonia (Abshire et al. 2017: 152–167), Britain and Ireland (Abshire et al. 2017), India beyond the Ganges, Serike and Sinae (Gusev and Stafeyev 2018), and West Africa (Filatova et al. 2019). We have demon-

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strated that additional clarity and understanding can be achieved by working directly with the coordinates, as opposed to merely relying upon visual comparison of old and modern maps in search of similarities. Under our iterative hybrid human-machine approach, the identified known objects mentioned by Ptolemy serve as reference points that help us place and identify previously unknown objects. The georeferencing task still remains very complex, due to the many source distortions and data compilation errors further compounded by the discrepancies between different manuscripts and editions of *Geography* that had been shown to be a significant contributing noise factor in its own right (Dilke 1987). This distortion noise is difficult to quantify in relation to Ptolemy's own errors. In our prior works, we have confirmed that the precision of our methods for numerical prediction of coordinates of the unknown points varied significantly region-to-region, in line with the uneven quality of data. In particular, we proposed and advocated a new hypothesis placing Ptolemy's Sinae province almost entirely in West Africa, along with some of the objects placed by Ptolemy in his India beyond the Ganges. In our paper on Ptolemy's Arabia (Abshire et al. 2020), we introduced object classification dividing all Ptolemy objects into four categories: *known* objects, *tentatively identified* objects, *unknown* objects (placed approximately), and *duplicates*. This classification will continue to serve us here.

While keeping our primary focus firmly on Ptolemy's *Geography*, we have been taking advantage of its rich context comprised of such works as (McCrindle 1927), the anonymous *Periplus of the Erythraean Sea* (Schoff 1912), the *Periplus of Hanno* (Schoff 1972), Herodotus (Marincola and A. de Sélincourt 1996), Pliny the Elder (Pliny 1855), the *Antonine Itinerary* (Cuntz 1929), *Tabula Peutingeriana* (Levi and Levi 1978), and the *Ravenna Cosmography* (Schnetz 1942). We will supply a review of other scientific literature relevant to the GIS analysis of Ptolemy's data in the old and modern contexts in the next section and revisit *Tabula Peutingeriana* in the section after that.

The fourth section of the paper will present our GIS analysis of Konrad Miller's reconstructed *Tabula Rogeriana* by Muhammad al-Idrisi (Miller 1929). The fifth section of our paper will discuss georeferencing a Hellenic Ptolemaic map by Nicephorus Gregoras (c. 1295 – 1360) from Vaticanus Urbinas Graecus 83 (15 c.) to Ptolemy's dataset.

The sixth section will be devoted to the analysis of object coordinates from the *Alfonsine tables* calculated for 1252 AD, the first year of the rule of King Alfonso X of Castile, by Isaac ben Sid and Jehuda ben Moses Cohen (1483), along with the coordinates of the extra objects added in the subsequent printings of the book.

In the seventh section of the work, we will talk about visualizing Ptolemy's one-sided globe of *oikouménē* using such modern GIS tools for development of 3D virtual globes as ESRI ArcGlobe, Google Earth, and Cesium. We will draw conclusions and discuss our plans for the future research in the eighth and final section of the paper.

Literature Review

The best complete modern translation of Claudius Ptolemy's *Geography* is the authoritative translation into German (Stückelberger and Grasshoff 2006). It comes with a Greek version of the original work printed alongside the German translation. The book is accompanied by a database of Ptolemy object coordinates in an electronic format. The database supplies a convenient Ptolemy object ID system that we have adopted in our work. Stückelberger and Grasshoff often suggest modern names for the Ptolemy objects. We have observed that many of their suggestions are

based on those from an earlier Latin translation of *Geography* by Müller (1883-1901) that also contains useful thorough lists of different spellings of Ptolemy's object names.

The only complete translation of Ptolemy's *Geography* into English (Ptolemy 1991) has long been known to be of bad quality (Diller 1935). Despite that, we have been using this source to come up with suitable English counterparts for the German object names. Given that the toponyms found in *Geography* have originated from many different ancient languages, we must admit that no set of English name assignments can be quite perfect. Berggren and Jones (2000) published an annotated modern English translation of the theoretical chapters of Ptolemy's *Geography*, and Diller (2009) translated its Book 8.

The atlases by Talbert (2000) and Åhlfeldt (2019) have served us as invaluable sources of information essential for initial identification of the known Ptolemy objects. Tsorlini (2011) developed an excellent catalog that covers Ptolemy's Mediterranean and Black Sea region and provides an original methodology for computation of modern coordinates. Gunn (2018) published a monograph on Ptolemy and discoveries in Asia.

Leo Bagrow provided important insights into the origin, dating, and nature of Ptolemy's *Geography* in his foundational works (Bagrow 1945), (Bagrow 1985). In particular, Bagrow pointed out that the extant manuscripts of *Geography* were created during the last centuries of the Byzantine Empire, and that Ptolemy's work had remained virtually unknown to authors for 1000 years after his death, with few exceptions. According to Bagrow, an anonymous author of a book on Armenia's geography (*Ashkharhatsuyts*), most likely, Anania Shirakatsi, relayed to us that Pappus of Alexandria (c. 290 – c. 350 AD) once reworked and abridged Ptolemy's *Geography*. Furthermore, Marcian of Heraclea (c. 4th c. AD) mentioned Ptolemy in his *Periplus of the Outer Sea* (*Periplus maris exteri*), yet said nothing about his maps. We mentioned these observations by Bagrow to prepare the reader for what we are about to present and illustrate next.

Tobler (1966) pioneered application of mathematical methods to georeferencing of old maps when he derived a set of equations to establish a relationship between the medieval Hereford map and an oblique Mercator projection. Numerous references to other works related to the GIS analysis of old maps can be found in the literature reviews incorporated in our prior works referenced in the introduction section above.

Tabula Peutingeriana and Ptolemy's Geography

Tabula Peutingeriana (Levi and Levi 1978) is an ancient *mappa mundi* that made it to modernity as a rectangular copy on parchment, 0.34 m tall and 6.82 m long, discovered in a library in the German city of Worms in 1494. To illustrate how different this map is from Ptolemy's, we have attempted to "georeference" a vertically stretched image of the 1887 *Conradi Milleri facsimile totum* of *Tabula Peutingeriana* to the full dataset of Ptolemy's object coordinates (Stückelberger and Grasshoff 2006) plotted in the Mercator projection so as to match the rectangular shape of *Tabula Peutingeriana.* An attempt was made to achieve a "good fit" in the area of the Mediterranean Sea. The resulting composite image is shown in Figure 1.

Major differences between the two data sets are immediately obvious to the viewer. This outcome is unsurprising, given the difficulties in georeferencing old *mappae mundi* encountered since (Tobler 1966). To say that *Tabula Peutingeriana* is in a very different projection would require us to assume that the very notion of projection was known to the map's anonymous author whose lifetime is dated approximately by the 4th or early 5th century AD, based on the presence of Constantinople (renamed from Byzantium c. 330 AD) and the prominence of Ravenna, the seat of the Western Roman Empire from 402 to 476, suggesting a 5th century revi-

sion (Levi and Levi 1967). In fact, it appears that if any numerical data related to Ptolemy's set was ever used in making this parchment at all, that could only be some known latitudes. The reader should bear in mind that the ancient astronomers could measure latitudes of objects on Earth with reasonably good precision at and before the time of Ptolemy. However, for the technical reasons explained in detail by Berggren and Jones (2000), this was not the case with the longitudes and longitude differences. Therefore, the longitude numbers were likely known not be trusted.

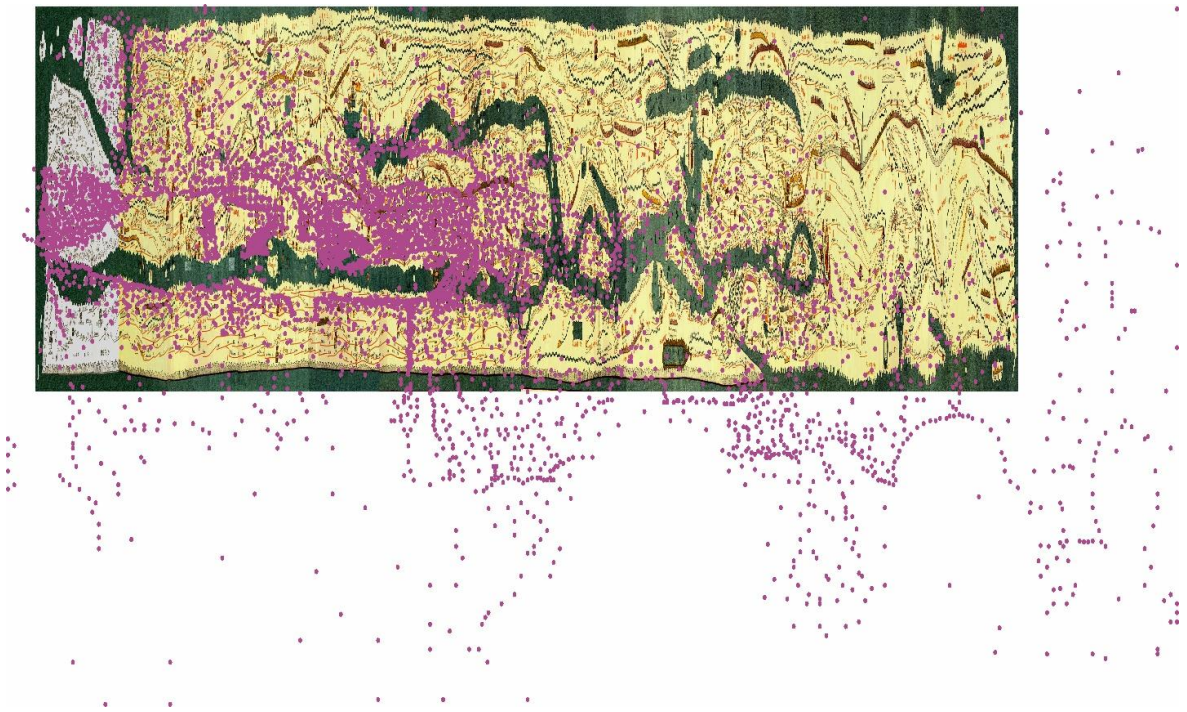

Figure 1: *Tabula Peutingeriana* vs. Ptolemy's *Geography* (the purple points).

And yet, *Tabula Peutingeriana* is far from useless when georeferencing Ptolemy's dataset to the modern map. That's because, unlike Ptolemy's maps, *Tabula Peutingeriana* captures the topology of the Roman road network and other trade route itineraries that it was apparently built from, with some peculiar "glitches". Let us illustrate this statement by inviting the reader to look closely at the West Africa fragment of *Tabula Peutingeriana* shown in Figure 2. (Konrad Miller's facsimile is used, for the sake of better readability.) Observe that both *Tingi* (modern Tangier, Ptol. *Tingis Caesaria*) and *Oppidum Novum* (*Oppido Novo*, Ptol. *Oppinum*) are shown twice, close to each other, as the mapmaker apparently accommodated data about different trade routes. Likewise, both *Baba* (*Babba Iulia Campestris* of Pliny the Elder (1855), Ptol. *Baba*) and *Baballaca* (Ptol. *Babiba* in Libya Interior, also written as *Babida* or *Babyla*) are shown. According to Rebuffat (1967), *Babba*'s existence was confirmed by an inscription found at the archaeological site of Thamusida (*Tamusida* of Ptolemy and *Tabula Peutingeriana*, also Ptol. *Tamusiga* as a likely duplicate located en route south, toward the phantom *Babiba*).

Another object to spot in Figure 2 is *Aquis Daticis*. On the modern map, it corresponds to the known ruins of Aquae Dacicae (lat=34.1466°, lon=-5.8007°). Ptolemy does not mention that place. However, *Baba* is now localized on the road between Aquae Dacicae and another well-known place, Volubilis (lat=34.0731°, lon=-5.5544°). Considering that the route from *Sala*

(the modern ruins of Chellah, near Sale) to *Gentiano* (Ptol. *Gontiana*) heads north in Ptolemy's coordinates, we can further conclude that *Baballaca-Babiba* is a mere duplicate of Baba, and the most likely location for *Babba Iulia Campestris* is then the vicinity of Bab Tissera (Filatova et al. 2019), near a mountain pass that would lead a traveler to Volubilis from either Aquae Dacicae, or *Gontiana*.

Another illuminating observation is that *Tabula Peutingeriana* swaps the locations of *Portum Magnum* (Ptol. *Portus Magnus*) and *Portum Divinum* (Ptol. *Deorum Harbor*) compared to Ptolemy. Moreover, the map shows them as if they were located on the road to *Calama* (Ptol. *Kelama*, modern Guelma in Algeria). *Calama* is misplaced by ~700 km, apparently because Guelma was once located near the old western border of the Roman Numidia, and then the borders of the Numidia province were redrawn when Western Numidia was annexed in 40 BC. Filatova et al. (2019) accepted Ptolemy's order of the ports, having taken into consideration that the modern name of one of the locations of interest, Mers El Kebir, literally means the same thing as *Portus Magnus*, i.e., the 'great port'.

In the next section of this paper, we will look at a "better, newer" map, Konrad Miller's reconstruction of *Tabula Rogeriana* by Muhammad al-Idrisi.

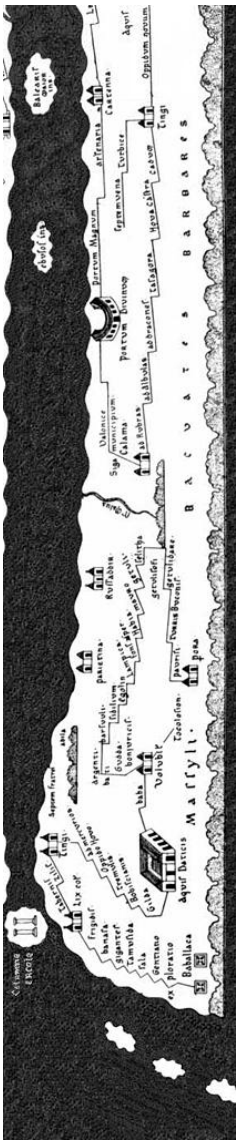

Figure 2: *Tabula Peutingeriana*, a fragment of Konrad Miller's facsimile (1887)

Tabula Rogeriana and Ptolemy's Geography

Miller (1929) produced a reconstruction of the famous *Tabula Rogeriana*, a world map by a prominent Arab scientist Muhammad al-Idrisi dated 1154 AD. The shape of the original *mappa mundi* drawn for King Roger II of Sicily was circular, but Miller followed the Mercator projection for latitudes between 28°N and 59°30'N, as he consolidated the map from 70 double-page spreads covering the 7×10 rectangular area. The southern portion of the map is compressed in the latitudinal direction, as the reader can readily observe from Figure 3 that shows our overlay georeferencing al-Idrisi's map to the complete Ptolemy dataset.

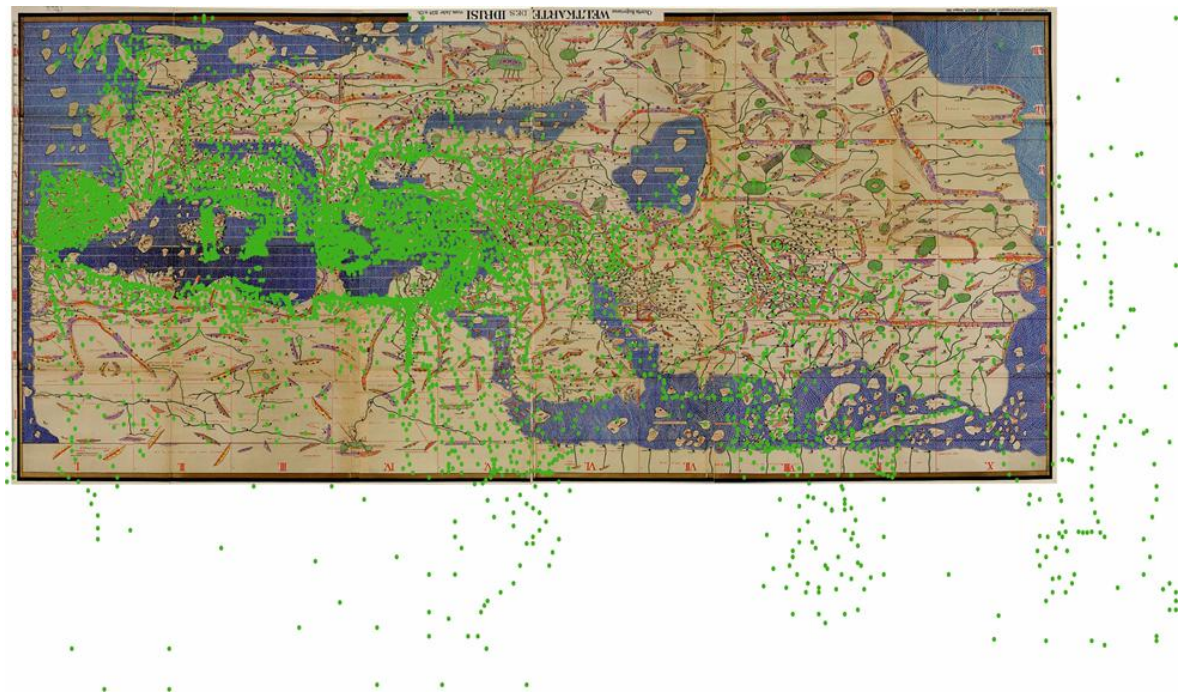

Figure 3: *Tabula Rogeriana* vs. Ptolemy's *Geography* (the green points).

With the best fit for the area of the Mediterranean Sea imposed, we can readily observe that not only the rest of al-Idrisi's map is narrower in the longitudinal direction than Ptolemy's *oikouménē*, but also big differences are noticeable in the Mediterranean area itself and around it. We can practically hear Ptolemy's criticism of "those who allotted the greatest part of the map to Europe in both longitude and latitude for the wealth of data being shown, and the least part in longitude to Asia and in latitude to Libya for the contrary" (Diller 2009). At the same time, al-Idrisi's map is notably free from another "deficiency" mistakenly criticized by Ptolemy, as it is not showing all of Europe, Africa, and Asia surrounded by ocean, as many other round *mappae mundi* did, along with *Tabula Peutingeriana*. This may be due to the influence of Ptolemy's work, as al-Idrisi adds new data on China eastward, past *Katigura* (Ptolemy's mysterious *Cattigara Sina)*, most likely a West African location, which Filatova et al. (2019) tentatively identified as the modern Settra Kru in Sinoe county, Liberia. Quoting (Ptolemy 1991), *The Hyrcanium sea, called also the Caspian, is surrounded on all sides by land and has the shape of an island; and we may say the same of the Indian sea, for with its gulfs, the Arabian, the Persian, the Gangetic, and that which is called the Great gulf, it is entirely shut in, like the Caspian, by land on all sides.*

We found that the roots of Ptolemy's error also present in al-Idrisi's map can be elucidated by reading *The Geography* by Strabo (64 or 63 BC – c. AD 24), an ancient Greek geographer

who offered a detailed discussion on how to interpret Homer's quote on the Ethiopians "sundered in twain" (Strabo 1917-1932). Strabo was a predecessor of Ptolemy who did subscribe to the correct view that the whole "known world" was surrounded by Oceanus. While the most obvious way to separate the Ethiopians into two parts is to do so by the Nile, an alternate explanation proposed by Crates involved dividing them by Oceanus, so they would end up "abiding both where Hyperion sets and where he rises." This helps explain sudden emergence of "fish-eating Aethiopians" in Ptolemy's Sinae, even though he had mistakenly placed it in East Asia. Neither *Tabula Peutingeriana* nor the map of Juan de la Cosa dated 1500 (Martín-Merás 2000) placed Sinae to the east from India. The latter fact is of special significance, given that Juan de la Cosa accompanied Christopher Columbus in his expeditions and demonstrated detailed knowledge of the African coast. Yet another, particularly striking explanation offered by Strabo is found in the following quote.

But Ethiopia may be divided in still another way, quite apart from this. For all those who have made coasting-voyages on the ocean along the shores of Libya, whether they started from the Red Sea or from the Pillars of Heracles, always turned back, after they had advanced a certain distance, because they were hindered by many perplexing circumstances, and consequently they left in the minds of most people the conviction that the intervening space was blocked by an isthmus; and yet the whole Atlantic Ocean is one unbroken body of water, and this is particularly true of the Southern Atlantic.

It definitely looks like Ptolemy has accepted this explanation, along with all other explanations that had the Ethiopians "sundered in twain" one way or another, and placed a very wide "isthmus" of unknown land south from the southernmost known land of Aethiopia, as if to directly counter Strabo's argument that "All those voyagers have spoken of the last districts to which they came in their voyagings as Ethiopic territory and have so reported them." Meanwhile, the concept of the *oikouménē* surrounded by ocean remained so pervasive that six out of ten extant copies of al-Idrisi's work have been reported to begin with a circular *mappa mundi* **not** mentioned in the text of his *Kitab Rujar* and looking like the image adopted from Wikipedia (2020) and shown in Figure 4.

In the next section, we will apply the same approach to georeference a known Hellenic Ptolemaic map by Nicephorus Gregoras from Vaticanus Urbinas Graecus 83 (15 c.) to Ptolemy's dataset. Whether or not al-Idrisi's work was influenced by Ptolemy's *Geography*, newer, better maps were coming!

Figure 4: Map from 'Alî ibn Hasan al-Hûfî al-Qâsimî's 1456 copy of al-Idrisi's *Kitab Rujar*.

Georeferencing a Hellenic Ptolemaic Map to Ptolemy's Dataset

Figure 5 shows the results of georeferencing a Hellenic Ptolemaic map of Europe by Nicephorus Gregoras from Vaticanus Urbinas Graecus 83 (early 15 c.) to Ptolemy's partial dataset. The points of different colors and shapes represent Ptolemy objects from several (not all) provinces of *oikouménē.*

If one discounts the anisotropy of the ancient medium and the manuscript-to-manuscript discrepancies in Ptolemy object coordinates, the match is perfect. This result is unsurprising. We can conclude that some medieval cartographers faithfully performed their tasks without contributing much extra information to the Ptolemy dataset, if any. The limitations on the size of the paper does not allow us to show a couple of similar results that we have obtained for old Ptolemaic maps.

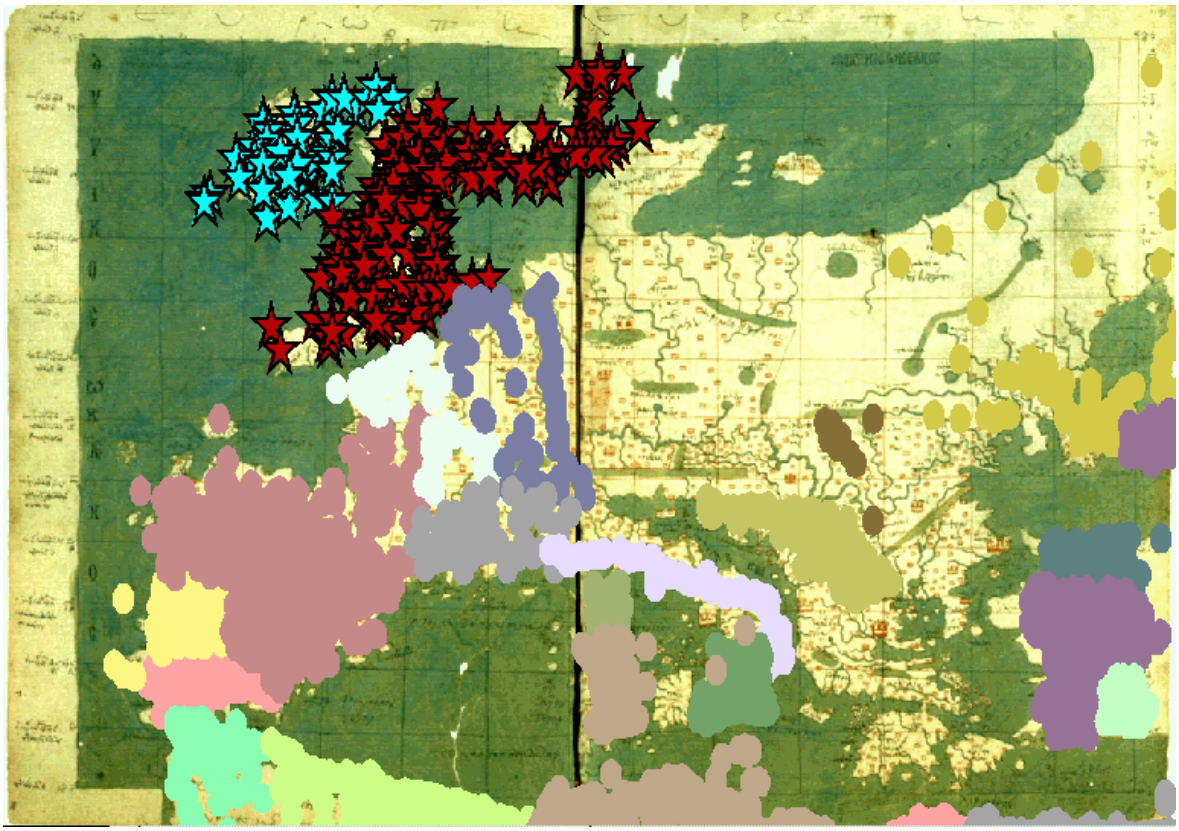

Figure 5: Hellenic Ptolemaic Map of Europe vs. Ptolemy's *Geography* (points of different colors and shapes correspond to different provinces of *oikouménē*).

The Alfonsine Tables and Ptolemy's Geography

The *Alfonsine tables* were primarily astronomical tables calculated for 1252 AD, the first year of the rule of King Alfonso X of Castile, León and Galicia, by Isaac ben Sid and Jehuda ben Moses Cohen (1483). On p. 6 of the above-referenced 1483 edition of the *Alfonsine tables*, we found a list of coordinates for 48 geographic objects, 2 of which turned out to be duplicates. We have listed all 48 objects, along with their Alfonsine, Ptolemaic, and modern coordinates in Table 1 of Appendix A. The 46 unique objects are mostly cities (43), 2 islands (Sardinia and Mallorca), and one mountain (apparently, Peñalara in Spain). 32 of the objects are located in Europe, 8 in Africa, and 6 in Asia, including *Xeanateh* (Xanadu), the capital of Kublai Khan since 1260. Out of the 32 European objects, 21 are located in the continental Italy, one more in Sicily (Palermo), and one is the Island of Sardinia, its coordinates matching the Ptolemy location of the Raging (*Mainomena*) Mountains in the middle of the island. The Alfonsine coordinates of Babylon match Ptolemy almost exactly (the longitude is off by 1 degree, 78° instead of 79°). The longitude of Damascus is reduced by 9 degrees, while the longitude of Jerusalem is reduced by 10 degrees and its latitude is rounded off to the nearest degree, compared to Ptolemy. The Alfonsine coordinates of Paris are those of Ptolemy's *Lutetia*, rounded off to the nearest degree. The coordinates of Rome match Ptolemy's *Geography* almost exactly. The coordinates of *Florentia* (Florence) happen to erroneously duplicate those of *Colonia* (Populonia). Given that the book was printed in Venice, it's amazing that the latitude of that city is off to the north by 5 degrees, perhaps, out of secrecy. Meanwhile, the latitude of Genoa is improved by a degree and a half. Overall, it looks like the author of the tables made a conscious attempt to correct Ptolemy's wrong turn of the Italian peninsula, this

impression pointing toward the Venetian, or otherwise Italian origin of the Italian part of the coordinate set from 1483.

Two very different tables of geographic objects were printed in two subsequent editions of the Alfonsine tables — (Ben Sid and ben Moses Cohen 1492) and (Ben Sid and ben Moses Cohen 1553). Unfortunately, in both cases, the longitudes were converted to hours and minutes from *Toletum* (Toledo, Spain) and the latitudes were rounded off to whole degrees. Our GIS analysis for these two newer sets of coordinates involved making the observation that the objects are listed in the same order in which they appeared in Ptolemy's *Geography* and recomputing the coordinates back to the Ptolemaic format using the following formula:

$$Alf.lon. = sign * (15 * hours + 0.25 * minutes) + 10°,$$

where *Alf. lon.* is an approximate equivalent of the original longitude, *sign* is the sign of the value given in the Alfonsine tables (1, if it's '+'; -1, if its '-'), *hours* and *minutes* are hours and minutes from Toledo, respectively, and 10° is the longitude of Toledo in Ptolemy's *Geography*. It's important to note that the 1553 edition added only 11 "new" objects to the set from the 1492 edition, and only two of those objects (Granada and Sevilla) were absent from the 1483 edition. The modern coordinates of the objects from the two newer editions were then established as listed in Table 2 of Appendix A, and most of them were mapped in Figure 6, alongside the modern locations of the objects listed in the 1483 edition. In Figure 7, we have additionally plotted a fit of the Alfonsine coordinates in the modern projection using ArcGIS so as to graphically illustrate the errors relative to the modern coordinates of the objects. The green circles in Figure 7 correspond to the georeferenced Alfonsine coordinates that were fitted to the modern projection, while the red circles connected to the green ones by the black error lines are placed at the modern coordinates of the objects.

Figure 6: Objects listed in the Alfonsine tables (1483 and 1492/1553).

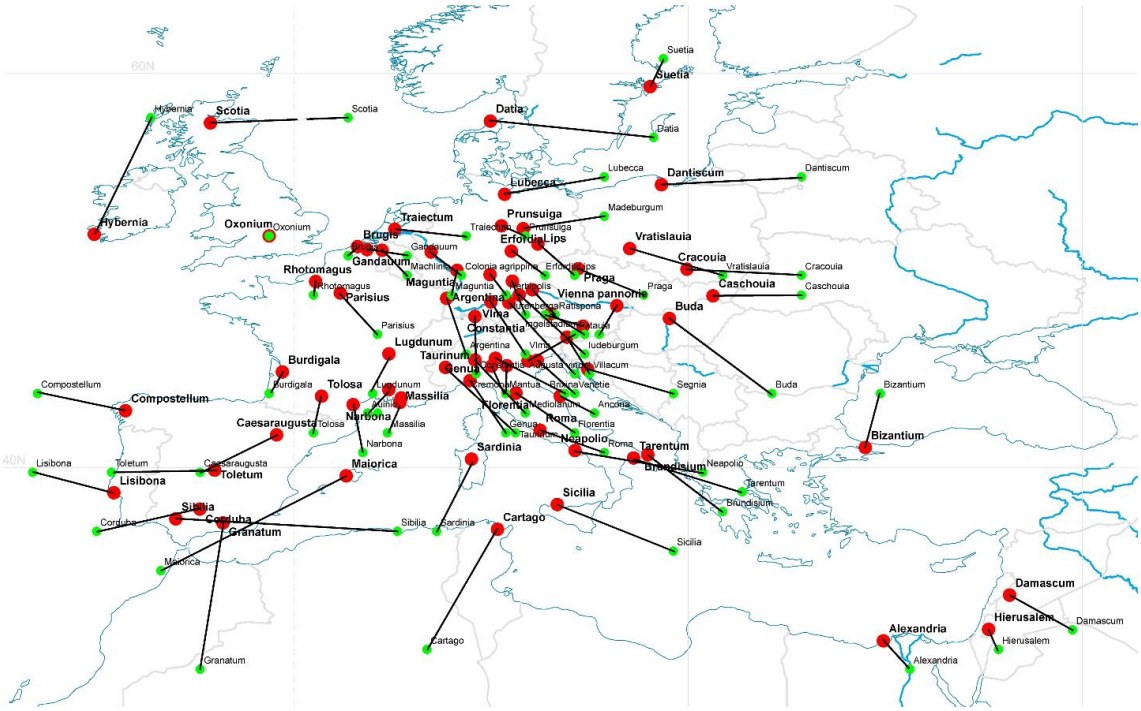

Figure 7: Errors in the Alfonsine tables.

It should be sufficiently obvious to the reader that the quality of longitude measurement was still poor in the late 15th – early 16th centuries, and the latitudes are far from being precise in many cases.

Visualizing Ptolemy's One-Sided Globe

We have plotted Ptolemy's dataset in ArcGlobe as shown in Figure 8 and began to convert our Ptolemy reconstructions to a KML file for Google Earth and Cesium (Gusev and Stafeyev 2020).

Figure 8: Ptolemy's One-Sided Globe of *oikouménē* in ArcGlobe.

Figure 9 shows the northern part of Ptolemy's Mauretania Tingitana province in *Cesium ion*.

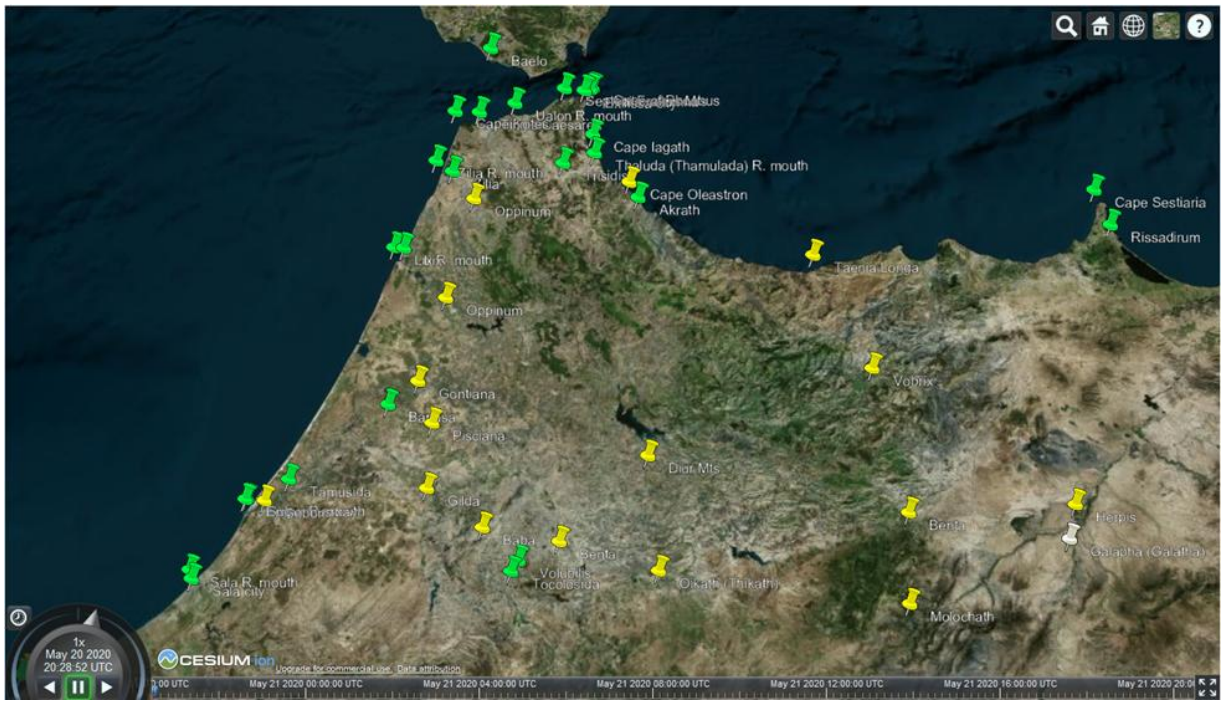

Figure 9: Ptolemy Globe in *Cesium ion*: The northern part of Mauretania Tingitana.

The green markers correspond to the known objects, the yellow ones mark the tentatively identified locations, and the white marker belongs to a suspected duplicate. An approximately placed unknown object would be displayed using a red marker. This part of Mauretania Tingitana is relevant to our earlier discussion of *Tabula Peutingeriana*, as the reader can see the route from the tentative location of Ptolemy's *Gontiana* (Souk el Arba?) to *Volubilis* via *Baba*.

Conclusions and Future Work

We have compared Claudius Ptolemy's quality of map-making to *Tabula Peutingeriana* (c. 4th or 5th century AD), *Tabula Rogeriana* by Muhammad al-Idrisi (12th c.), and three editions of the *Alfonsine tables* (13th c.; published in 1483, 1492, and 1553). Our GIS analysis visually demonstrates that his achievements remained largely unsurpassed until the end of the 15th century AD. This result helps emphasize the special role of Ptolemy's *Geography* as an invaluable tool for better understanding of development of civilization in antiquity and the middle ages. In the near future, we expect to expand our iterative GIS reconstruction of Ptolemy's "known world" to other regions, such as East Africa, while continuing to transfer our existing knowledge to the formats for convenient visualization in the modern GIS tools in order to make our results more accessible to the history of cartography researchers and the general public.

References

Abshire, C., Durham, A., Gusev, D. A., and Stafeyev, S. K. (2017). Ptolemy's Britain and Ireland: A New Digital Reconstruction. In *Proceedings of the 28th International Cartographic Conference*. Washington, DC: International Cartographic Association.

Abshire, C., Gusev, D. A., Papapanagiotou I., and Stafeyev, S. K. (2016). A Mathematical Method for Visualizing Ptolemy's India in Modern GIS Tools. *e-Perimetron* 11(1): 13–34.

Abshire, C., Gusev, D. A., Stafeyev, S. K., and Wang, M. (2020). Enhanced Mathematical Method for Visualizing Ptolemy's Arabia. *e-Perimetron* 15(1): 1–25.

Abshire, C., Gusev, D. A., and Stafeyev, S. K. (2017). The Fertile Crescent in Ptolemy's "Geography": A New Digital Reconstruction for Modern GIS Tools. In *Proceedings of the 12th ICA Conference on Digital Approaches to Cartographic Heritage* (pp. 152–167). Venice, Italy: International Cartographic Association.

Åhlfeldt, J. (2019). *Digital Atlas of the Roman Empire*. In digital form, https://dh.gu.se/dare/

Bagrow, L. (1945). The Origin of Ptolemy's Geographia. *Geografiska Annaler* 27: 318–387.

Bagrow, L. (1985). *History of Cartography*, 2nd ed., ed. R. A. Skelton. Chicago: Precedent Books.

Ben Sid, I. and J. ben Moses Cohen (1483). *Tabulae Alfonsi (h. 1-22v). Tabulae magistri Johannis de Lineris (h. 23-48v)*. Venice, Italy: Erhard Ratdolt. In digital form, https://www.wdl.org/ru/item/7326/view/1/11/

Ben Sid, I. and J. ben Moses Cohen (1492). *Tabule astronomice Alfonsi Regis*. Venice, Italy: Iohannis Hamman de landoia dictus Hertzog.

Ben Sid, I. and J. ben Moses Cohen (1553). *Divi Alphonsi romanorum et hispaniarum regis astronomicae tabulae*. Paris, France: Ex officina Christiani Wecheli.

Berggren J. and A. Jones (2000). *Ptolemy's Geography: An annotated translation of the theoretical chapters*. Princeton, NJ: Princeton University Press.

Cuntz, O. (1929). *Itineraria Romana, Vol. 1: Itineraria Antonini Augusti et Burdigalense*. Leipzig, Germany: Teubner.

Dilke, O.A.W. (1987). The Culmination of Greek Cartography in Ptolemy. In *The History of Cartography*, Vol. 1, ed. J.B. Harley and D. Woodward (pp. 177-200). Chicago, IL: University of Chicago Press.

Diller, A. (1935). Review of Stevenson's translation. *Isis* 22 (2): 533−539. In digital form, http://penelope.uchicago.edu/Thayer/E/Journals/ISIS/22/2/reviews/Stevensons_Ptolemy*.html

Diller, A. (2009). Ptolemy GEOGRAPHY Book 8. *DIO* 5: 1−42. In digital form, http://www.dioi.org/vols/w50.pdf

Filatova, L. M., Gusev, D. A., and Stafeyev, S. K. (2019). Iterative Reconstruction of Ptolemy's West Africa Using Modern GIS Analysis. In *Proceedings of the 14th ICA Conference on Digital Approaches to Cartographic Heritage* (pp. 204–236). Thessaloniki, Greece: The International Cartographic Association.

Gunn, G. C. (2018). *Overcoming Ptolemy: The Revelation of an Asian World Region.* Lanham, MD: Lexington Books.

Gusev, D. A., and Stafeyev, S. K. (2018). GIS Analysis and Digital Reconstruction of Ptolemy's India beyond the Ganges, Serike and Sinae. In *Proceedings of the 13th ICA Conference on Digital Approaches to Cartographic Heritage* (pp. 152–178). Madrid, Spain: International Cartographic Association.

Gusev, D. A., and Stafeyev, S. K. (2020). Ptolemy Globe. In digital form, https://ptolemy-globe.glitch.me/

Levi, A., and Levi, M. (1967). *Itineraria picta: Contributo allo studio della Tabula Peutingeriana.* Rome, Italy: Bretschneider.

Levi, A., and Levi, M. (1978). *La Tabula Peutingeriana*. Bologna, Italy: Edizioni Edison. In digital form, https://www.hs-augsburg.de/~harsch/Chronologia/Lspost03/Tabula/tab_intr.html

Livieratos, E. (2006). On the Study of the Geometric Properties of Historical Cartographic Representations. *Cartographica* 41(2): 165-175.

Marincola, J. M. and A. de Sélincourt (1996). *Herodotus: The Histories*. London, The UK: Penguin Classics.

Martín-Merás, L. (2000). La carta de Juan de la Cosa: interpretación e historia. *Monte Buciero* 4: 71–85.

McCrindle, J. W. (1927). *Ancient India as described by Ptolemy*. New Delhi: Munshiram Manoharlal Publishers Pvt. Ltd.

Miller, K. (1929). *Reconstitution of the Map Tabula Rogeriana by Arab Geographer Al Idrisi*. Stuttgart, Germany: Konrad Miller. In digital form,

https://commons.wikimedia.org/wiki/File:Tabula_Rogeriana_1929_copy_by_Konrad_Miller.jpg

Müller, K. (1883-1901). *Klaudiou Ptolemaiou Geographikê hyphêgêsis: Claudii Ptolemaei Geographia*, 2 vols. Paris: Firmin-Didot.

Pliny the Elder (1855). *Natural History*, ed. J. Bostock and H.T. Riley. London: Taylor and Francis.

Ptolemy, C. (1991). *Claudius Ptolemy, The Geography*, Translated and edited by E. L. Stevenson. New York, NY: Dover Publications, Inc.

Rebuffat, R. (1967). Les erreurs de Pline et la position de Babba Iulia Campestris. *Antiquites Africaines* 1: 31-57.

Schnetz, J. (1942) *Itineraria Romana, Vol. 2: Ravennatis Anonymi cosmographia et Guidonis geographica*. Stuttgart, Germany: Teubner.

Schoff, W. H. (1912). *Periplus of the Erythraean Sea: Travel and Trade in the Indian Ocean by a Merchant of the First Century*, Translated from the Greek and Annotated. New York, NY: Longmans, Green, and Co.

Schoff, W. H. (1972). The Periplus of Hanno: A Voyage of Discovery Down the West African Coast, by a Carthaginian Admiral of the Fifth century B.C., Translated from the Greek. *Paideuma: A Journal Devoted to Ezra Pound Scholarship* 1(2): 224-226.

Strabo (1917-1932). *The Geography*, Greek texts with facing English translation by H. L. Jones. Cambridge, MA: Loeb Classical Library, Harvard University Press. In digital form, https://penelope.uchicago.edu/Thayer/E/Roman/Texts/Strabo/home.html

Stückelberger A. and G. Grasshoff (2006). *Klaudios Ptolemaios: Handbuch der Geographie, Griechisch-Deutsch*. Basel: Schwabe Verlag.

Talbert, R. J. A. (ed.) (2000). *Barrington Atlas of the Greek and Roman World*. Princeton, NJ: Princeton University Press.

Tobler, W. R. (1966). Medieval Distortions, the Projections of Ancient Maps. *Annals of the Association of American Geographers* 56(2): 351-360.

Tsorlini, A. (2011). *Claudius Ptolemy "Geōgrafikē Yfēgēsis" (Geographia): digital analysis, evaluation, processing and mapping the coordinates of Greece, the Mediterranean and the Black Sea, based on 4 manuscripts and 15 printed editions, from Vaticanus Urbinas Gr. 82 (13th cent.) until today : the new Catalogue "GeoPtolemy-θ"*. Ph.D. thesis. Aristotle University of Thessaloniki: School of Rural and Surveying Engineering. In digital form, http://digital.lib.auth.gr/record/128272

Vaticanus Urbinas Graecus 83 (15 c.) 20r-116r: Κλαουδίου ΠΤΟΛΕΜΑΙΟΥ γεωγραφικῆς ὑφηγήσεως τὸ δεύτερον. In digital form, http://www.mss.vatlib.it/guii/scan/link1.jsp?fond=Urb.gr.

Wikipedia (2020). *Muhammad al-Idrisi*. In digital form, https://en.wikipedia.org/wiki/Muhammad_al-Idrisi

Appendix A. Tables of Objects from the *Alfonsine Tables* (1483 and 1492/1553)

Table 1. 48 Objects from the Alfonsine Tables (1483).

Alfonsine Name	*Ptolemy Name*	*Modern Name*	*Alf. Lon.*	*Alf. Lat.*	*Ptol. Lon.*	*Ptol. Lat.*	*Mod. Lat.*	*Mod. Lon.*
Constantino-ple	Byzantium	Istanbul	58° 40'	43° 40'	56° 00'	43° 05'	41.0085	28.9800
Cialia	Caelia	Ceglie Messapica	36° 00'	39° 00'	42° 10'	40° 15'	40.6444	17.5182
Alosandria	Alexandria	Alexandria	51° 20'	31° 00'	60° 30'	31° 00'	31.1946	29.9041
Damasai	Phatnitic mouth of the Nile	Damietta, former Tamiathis	54° 00'	32° 00'	62° 30'	31° 10'	31.4165	31.8133
Horaclia	Herakleopolis Mikra	Tall Tulaym (Tell Belim)	54° 40'	31° 00'	63° 20'	31° 00'	30.9784	32.1746
Egyptus	Koptos	Qift	59° 00'	30° 09'	62° 30'	26° 10'	25.9967	32.8159
Babilonia	Babylon	Tell Babil	78° 00'	35° 00'	79° 00'	35° 00'	32.5420	44.4212
Jerusalem	Aelia Capitolina = Hierosolyma	Jerusalem	56° 00'	32° 00'	66° 00'	31° 40'	31.7767	35.2342
Tur	Tyros	Tyre	57° 00'	33° 00'	67° 00'	33° 20'	33.2693	35.1959
Panormus	Panormus	Palermo	37° 30'	38° 16'	37° 00'	37° 00'	38.1123	13.3559
Damascus	Damaskos	Damascus	60° 00'	33° 00'	69° 00'	33° 00'	33.5114	36.3074
Tolosa	Tolosa	Toulouse	23° 47'	49° 00'	20° 10'	44° 15'	43.6147	1.3977
Parisius	Lutetia	Paris	23° 00'	48° 00'	23° 30'	48° 10'	48.8549	2.3475
Marsilia	Massilia	Marseille	27° 30'	44° 00'	24° 30'	43° 05'	43.2975	5.3746
Cremona	Cremona	Cremona	31° 00'	45° 00'	32° 00'	43° 40'	45.1350	10.0293
Bononia	Bononia	Bologna	33° 35'	44° 00'	33° 30'	43° 30'	44.4946	11.3424
Roma	Roma	Rome	36° 42'	41° 50'	36° 40'	41° 40'	41.8926	12.4843
Corduba	Corduba	Córdoba	9° 00'	37° 00'	9° 20'	38° 05'	37.8848	-4.7764
Xeanateh	-	Xanadu, Shangdu	153° 00'	10° 45'	-	-	42.3595	116.1805
Cartago	Karchedon	Carthage ruins (Byrsa)	27° 00'	32° 00'	34° 50'	32° 40'	36.8524	10.3236

Meca	Macoraba	Mecca	67° 00'	21° 00'	73° 20'	22° 00'	21.4234	39.8250
Insula Sardinia	Raging (Mainomena) Mts., Sardinia	Monte Rasu	31° 00'	38° 00'	31° 00'	38° 00'	40.4214	9.0047
Mediolanum	Mediolanium	Milan	30° 20'	44° 40'	30° 40'	44° 15'	45.4582	9.1810
Forlinium	-	Forli	33° 36'	45° 24'	-	-	44.2227	12.0407
Neapolis	Neapolis	Naples	36° 38'	40° 20'	40° 00'	40° 55'	40.8509	14.2581
Ancona	Ancona	Ancona	36° 00'	44° 30'	36° 30'	43° 40'	43.6253	13.5103
Firmium	Firmum	Fermo	36° 00'	43° 42'	37° 30'	42° 55'	43.1611	13.7168
Pisae	Pisae	Pisa	33° 00'	43° 00'	33° 30'	42° 45'	43.7223	10.4020
Mantua	Mantua	Mantua	31° 39'	45° 15'	32° 45'	43° 40'	45.1600	10.7968
Ferraria	-	Ferrara	32° 00'	[45° 15']	-	-	44.8381	11.6198
Salernum	Salernum	Salerno	38° 00'	41° 00'	40° 00'	40° 20'	40.6780	14.7659
Osama	Icosium	Algiers	19° 00'	35° 00'	18° 00'	33° 00'	36.7841	3.0579
Brandusui	Brundisium	Brindisi	40° 30'	41° 20'	42° 30'	39° 40'	40.6411	17.9469
Padua	Patavium	Padua	32° 30'	45° 24'	32° 50'	44° 30'	45.4115	11.8790
Tullectum	Toletum	Toledo	11° 00'	39° 54'	10° 00'	41° 00'	39.8585	-4.0252
Monte Pesulanium	-	Peñalara Mtn.	14° 30'	43° 00'	-	-	40.8501	-3.9562
Colonia	Populonium	Populonia	33° 25'	42° 30'	33° 30'	42° 30'	42.9881	10.4897
Axinus	Auxume	Aksum, Axum	27° 30'	0° 00'	65° 30'	11° 00'	14.1319	38.7192
Capua	Capua	Capua	37° 15'	40° 30'	40° 00'	41° 10'	41.1061	14.2130
Tunix	Catadas R. mouth	Tunis / Oued Miliane	29° 00'	37° 00'	34° 50'	32° 30'	36.8065 36.7716	10.1815 10.2921
Florentia	Florentia	Florence	33° 25'	42° 30'	33° 50'	43° 00'	43.7692	11.2599
Venetie	Atrianus R. mouth	Venice / Po di Levante	35° 20'	52° 20'	34° 00'	44° 30'	45.4340 45.0505	12.3390 12.3638
Maiorica	Palma	Palma de Mallorca	13° 26'	36° 00'	16° 10'	39° 15'	39.5696	2.6502
Cialia (duplicate of #2)	Caelia	Ceglie Messapica	36° 00'	39° 00'	42° 10'	40° 15'	40.6444	17.5182
Panurmus (duplicate of #10)	Panormus	Palermo	37° 30'	38° 16'	37° 00'	37° 00'	38.1123	13.3559
Janua	Genua	Genoa	30° 30'	44° 00'	30° 00'	42° 50'	44.4067	8.9333
Verona	Verona	Verona	32° 14'	44° 30'	33° 00'	44° 00'	45.4390	10.9944
Civitas Sanpie (*conj.* Candie)	Kydonia	Chania	52° 00'	36° 00'	52° 45'	35° 00'	35.5182	24.0230

Table 2. 73 Objects from the Alfonsine Tables (1492 and 1553).

Alfonsine Name	*Ptolemy Name*	*Modern Name*	*Alf. Lon.*	*Alf. Lat.*	*Ptol. Lon.*	*Ptol. Lat.*	*Mod. Lat.*	*Mod. Lon.*
Hybernia	Hibernis	Church Island in Lough	12	59	11	58.17	51.8351	-10.1304

		Currane, Ireland						
Scotia	Tuesis estuary (*conj*. Nuesis)	Ness R. mouth	22	59	27	59	57.4950	-4.2360
Oxonium		Oxford /	18	53			51.7520	-1.2577
	Corinium	Cirenchester			18	54.17	51.7187	-1.9619
Compostellum	Aquae Calidae	Santiago de Compostela	6.25	45	6.33	44.33	42.8806	-8.5447
Lisibona	Olisipo	Lisbon	6	41	5.17	40.25	38.7104	-9.1324
Toletum	Toletum	Toledo	10	41	10	41	39.8585	-4.0252
Corduba	Corduba	Córdoba	9.25	38	9.33	38.08	37.8848	-4.7764
Caesaraugusta	Caesaraugusta	Zaragoza	14.5	41	14.25	41.5	41.6522	-0.8775
Rhotomagus	Rotomagus	Rouen	20.25	50	20.17	50.33	49.4398	1.0985
Parisius	Lutetia	Paris	23.5	48	23.5	48.17	48.8549	2.3475
Lugdunum	Lugdunum Metropolis	Lyon	23.25	45	23.25	45.83	45.7597	4.8198
Burdigala	Burdligala	Bordeaux	18	45	18	45	44.8477	-0.5830
Auinio	Avennio	Avignon	23	44	23	44	43.9515	4.8068
Tolosa	Tolosa	Toulouse	20.25	43	20.17	44.25	43.6147	1.3977
Vienna prouintie	Aquae Sextiae	Aix-en-Provence	23.5	44	24.5	43.67	43.5319	5.4474
Massilia	Massilia	Marseille	24	43	24.5	43.08	43.2975	5.3746
Brugis	Frudis R. mouth	Bruges /	22	52			51.2093	3.2247
		Canche R. mouth; tributary: Ternoise R.			21.75	52.33	50.5364	1.6038
Gandauum	Canduum	Ghent	25	52	33	51.33	51.0548	3.7169
Traiectum	-	Utrecht	28	53	-	-	52.0907	5.1214
Colonia agrippina	Colonia Agrippinensis	Cologne	27.75	51	27.67	51.5	50.9413	6.9577
Machlinia	-	Mechelen	25	51	-	-	51.0259	4.4775
Maguntia	Mogontiacum	Mainz	27.25	50	27.33	50.25	50.0013	8.2679
Herbipolis	Artaunum	Würzburg	30	50	30.17	50	49.7913	9.9534
Argentina	Argentorate	Strasbourg	28	47	27.75	48.33	48.5828	7.7480
Constantia	Drusomagus	Konstanz	28.5	46	31.5	46.08	47.6633	9.1762
Augusta vindel.	Augusta Vindelicorum	Augsburg	33.5	46	32.5	46.33	48.3663	10.8968
Datia	Cimbrian peninsula "next further north"	Denmark (Hirtshals)	37.5	58	38.67	58.17	57.5869	9.9669
Suetia	Scandia N	Sweden (Stockholm)	38	62	44.5	58.5	59.3293	18.0686
Lubecca	Treva	Lübeck	35	56	33	55.67	53.8669	10.6880
Dantiscum	Vistula R. mouth	Gdańsk /	45	56	45	56	54.3520	18.6466
		Martwa Wisła					54.4104	18.6607

Prunsuiga	Tulisurgium	Braunschweig	31	53	32.67	53.33	52.2659	10.5267
Madeburgum	Mesuium	Magdeburg	35	54	35.5	53.83	52.1205	11.6276
Erfordia	Amisia	Erfurt	32	51	31.5	51	50.9787	11.0328
Lips	Luppia	Leipzig	33.5	51	34.5	52.75	51.3396	12.3731
Ingelstadium	Abiluum (Abilunum)	Ingolstadt	32	49	35.33	47.33	48.7651	11.4237
Nurenberga	Segodunum	Nuremberg	31	49	31.5	49	49.4539	11.0773
Ratispona	Devona	Regensburg	32.5	49	32.5	48.75	49.0167	12.1021
Vlma	Alcimoennis	Ulm	31	47	32.5	47.5	48.3985	9.9925
Praga	Casurgis	Prague	37	50	39.25	50.17	50.0876	14.4212
Vratislauia	Viritium	Wrocław	41	51	41	54.5	51.1104	17.0372
Cracouia	Carrodunum	Kraków	45	51	42.67	50.5	50.0615	19.9378
Caschouia	Setuia	Košice	45	50	42.5	50	48.7138	21.2580
Buda	Aquincum	Budapest	43.5	45	43	47.5	47.5646	19.0494
Segnia	Senia	Senj	38.5	45	39	44.67	44.9894	14.9036
Vienna pannonie	Vindobona	Vienna	34.75	48	37.75	46.33	48.2107	16.3727
Patauia	Patavium	Padua	33.5	48	32.83	44.5	45.4115	11.8790
Saltzeburgum	Bedaium	Salzburg / Seebruck	34	48	34.25	46.25	47.7993 47.9334	13.0439 12.4768
Iudeburgum	Idunum	Judenburg	34	47	35.17	45.5	47.1686	14.6606
Villacum	Santicum	Villach	34.25	46	36	45.5	46.6133	13.8443
Brixina	Brixia	Brescia	33	45	32.5	44.17	45.5399	10.2258
Venetie	Atrianus R. mouth	Venice/ Po di Levante	33.5	45	34	44.5	45.4340 45.0505	12.3390 12.3638
Ancona	Ancona	Ancona	34.5	44	36.5	43.67	43.6253	13.5103
Roma	Roma	Rome	35	42	36.67	41.67	41.8926	12.4843
Tarentum	Tarentum	Taranto	42	40	42.17	40	40.4738	17.2330
Brundisium	Brundisium	Brindisi	41	39	42.5	39.67	40.6411	17.9469
Neapolio	Neapolis	Naples	40	41	40	40.92	40.8509	14.2581
Florentia	Florentia	Florence	33.5	43	33.83	43	43.7692	11.2599
Mediolanum	Mediolanium	Milan	31	44	30.67	44.25	45.4582	9.1810
Taurinum	Augusta Taurinorum	Turin	30.5	43	30.5	43.67	45.0750	7.6846
Genua	Genua	Genoa	30	43	30	42.83	44.4067	8.9333
Sardinia	Raging (Mainomena) Mts., Sardinia	Monte Rasu	26.5	38	31	38	40.4214	9.0047
Sicilia	Panormus	Palermo	38.5	37	37	37	38.1123	13.3559
Maiorica	Palma	Palma de Mallorca	12.5	36	16.17	39.25	39.5696	2.6502
Granatum	Iliberri	Granada	14.5	31	11	37.67	37.1822	-3.5997
Narbona	Narbo colonia	Narbonne	22.75	42	21.5	43.25	43.1855	3.0046
Sibilia	Hispalis	Seville	24.5	38	7.25	37.83	37.3843	-5.9899
Cartago	Karchedon	Carthage	26	32	34.83	32.67	36.8524	10.3236

		ruins (Byrsa)						
Mantua	Mantua	Mantua	30	45	32.75	43.67	45.1600	10.7968
Cremona	Cremona	Cremona	30	45	32	43.67	45.1350	10.0293
Bizantium	Byzantium	Istanbul	49	45	56	43.08	41.0085	28.9800
Alexandria	Alexandria	Alexandria	50.5	31	60.5	31	31.1946	29.9041
Hierusalem	Aelia Capitolina = Hierosolyma	Jerusalem	55	32	66	31.67	31.7767	35.2342
Damascum	Damaskos	Damascus	58.75	33	69	33	33.5114	36.3074